Modeling and Simulation.

Practical Assignments 2.

For all of the assignments you are supposed to write a program in C or C++ with suitable "general purpose" routines and a main routine to solve the particular problem in the assignment. Write a concise but comprehensive report to describe the results of your experiments with the code. Use double-precision floating point variables. The code should run on the student machines. Use the server machine 'deze.science.uva.nl' as the standard.
The assignments can again be made in pairs. Submission deadline is Wednesday, Feb. 23.

1 Differentiation and roots

Assignment 1

Write a set of routines to perform numerical differentiation on a given function, given a parameter value x and an increment h. Use right-hand and central differencing.
Now use these routines to find the derivative of $\sin(x)$ for $x = \pi/3, 100\pi + \pi/3, 10^{12}\pi + \pi/3$. Experiment with the value of h to find the most accurate result in each case.
For what value of h do you find the most accurate result in each case and what is that result.

Assignment 2

Use the bisection method to find a zero of the function

$$f(x) = x\sin(x) - 1$$

on $x \in [0, 2]$.
Record how many steps you need and at what rate the error decreases.

Assignment 3

Calculate the value of $\sqrt{2}$ using the bisection method.
Record how many steps you need and at what rate the error decreases. Now do the same experiment using the "false position" method and Newton-Raphson.

Assignment 4

Use the Newton-Raphson method to compute the zeros of

$$f(x) = x^2 - x + 2$$

and

$$f(x) = x^3 - 3x - 2$$

and

$$f(x) = (x^2 + 1)(x - 4)$$

How do you find a suitable starting value for x? Would it help to do the first few iterations using the bisection or regula falsi methods?

2 Integration

For the assignments in this section, you may want to look at
http://en.wikipedia.org/wiki/Numerical_integration

Assignment 5

Write three routines to integrate a function over a specified interval, one using the rectangle rule, one using the trapezoidal rule, the third using Simpson and finally one using a two-point Gauss integration. All four should allow you to specify how many subdivisions should be used on the interval.
Test the accuracy of those routines for the following integrals:

$$\int_0^1 e^{-x} dx$$

$$\int_0^2 xe^{-x} dx$$

$$\int_0^{20} xe^{-x} dx$$

$$\int_0^{200} xe^{-x} dx$$

$$\int_0^{8\pi} \sin(x) dx$$

Can you also calculate the following integral (you should be able to do it by hand):

$$\int_0^2 x^{-0.5} dx$$

Assignment 6

Now try to create a routine that integrates a function with a desired accuracy, by computing the integral with different numbers of sample points, comparing the results and increasing the number of sample points if the required accuracy is not attained. Think carefully about how you should define the accuracy. Do this for each of of the above four methods and investigate how many refinements you need for each to reach a specified accuracy.
Test your routines on some suitably challenging integrals.